LAU
School of Arts and Sciences

LEBANESE AMERICAN UNIVERSITY
Department of Computer Science and Mathematics

MTH101 – Calculus I
Spring 2015
Exam 1
(February 25, 2015)

NAME: Answer Key **ID:**

Duration: **60 minutes**

Instructor: Ms. Liwa Sleiman

This exam is comprised of 5 problems. Answer the questions in the space provided for each problem. If more space is needed, use the back of the page. Make sure to justify all your answers.

Problem	Grade points	
I	25	
II	15	
III	10	
IV	10	
V	15	
Total	**75**	

I. a) (5 pts.) Find the equation of the line passing through the points A(3,4) and B(0,-2). 1

$m = \frac{y_B - y_A}{x_B - x_A} = \frac{-2-4}{0-3} = \frac{-6}{-3} = 2$ 1

1 $y - y_A = m(x - x_A)$

1 $y - 4 = 2(x-3)$

$y - 4 = 2x - 6$

1 $\boxed{y = 2x - 2}$

$\boxed{OR}$

$y = mx + b$ 1

$m = 2$, $b = -2$ = y-intercept B(0,-2) 1

$\boxed{y = 2x - 2}$ 1

b) (5 pts.) Find the center and radius of the circle

$x^2 + y^2 + 6x - 8y = 5$

$x^2 + 6x + y^2 - 8y = 5$

$\underline{x^2 + 6x + 3^2}$ (1/2) $+ \underline{y^2 - 8y + 4^2}$ (1/2) $= 5 + 3^2 + 4^2$ (1/2)

$(x+3)^2$ (1/2) $+ (y-4)^2$ (1/2) $= 5 + 9 + 16$
$= 30$ (1/2)

1 Center $(-3,4)$

1 Radius $R = \sqrt{30}$

c) (5 pts.) Consider the functions

$f(x) = 3x - 4,\ \ g(x) = x^2 + 1,\ \text{ and }\ h(x) = \frac{1}{x}$

Write a formula for $f \circ g \circ h$.

$f \circ g \circ h(x) = f[g(h(x))]$ 1 $= f[g(\frac{1}{x})]$ 1 $= f[(\frac{1}{x})^2 + 1]$

$= f[\frac{1}{x^2} + 1]$ 1 $= 3(\frac{1}{x^2} + 1) - 4$ 1 $= \frac{3}{x^2} + 3 - 4$

$= \boxed{\frac{3}{x^2} - 1}$ 1

d) (5 pts.) Find the domain of $p(x) = \frac{2x-7}{4-\sqrt{x+6}}$

2 conditions

everything inside the $\sqrt{\ }$ is ≥ 0 1

$x + 6 \geq 0$

$\boxed{x \geq -6}$ (1/2)

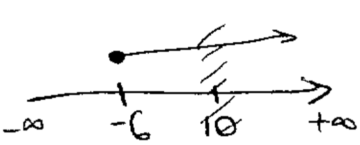
denominator $\neq 0$

1 $4 - \sqrt{x+6} \neq 0$

$4 \neq \sqrt{x+6}$

$16 \neq x + 6$

(1/2) $\boxed{x \neq 10}$



$D = [-6, 10) \cup (10, +\infty)$ 2

e) (5 pts.) Determine whether the function $y = x^5 - 1$ is even, odd, or neither. Justify your answer.

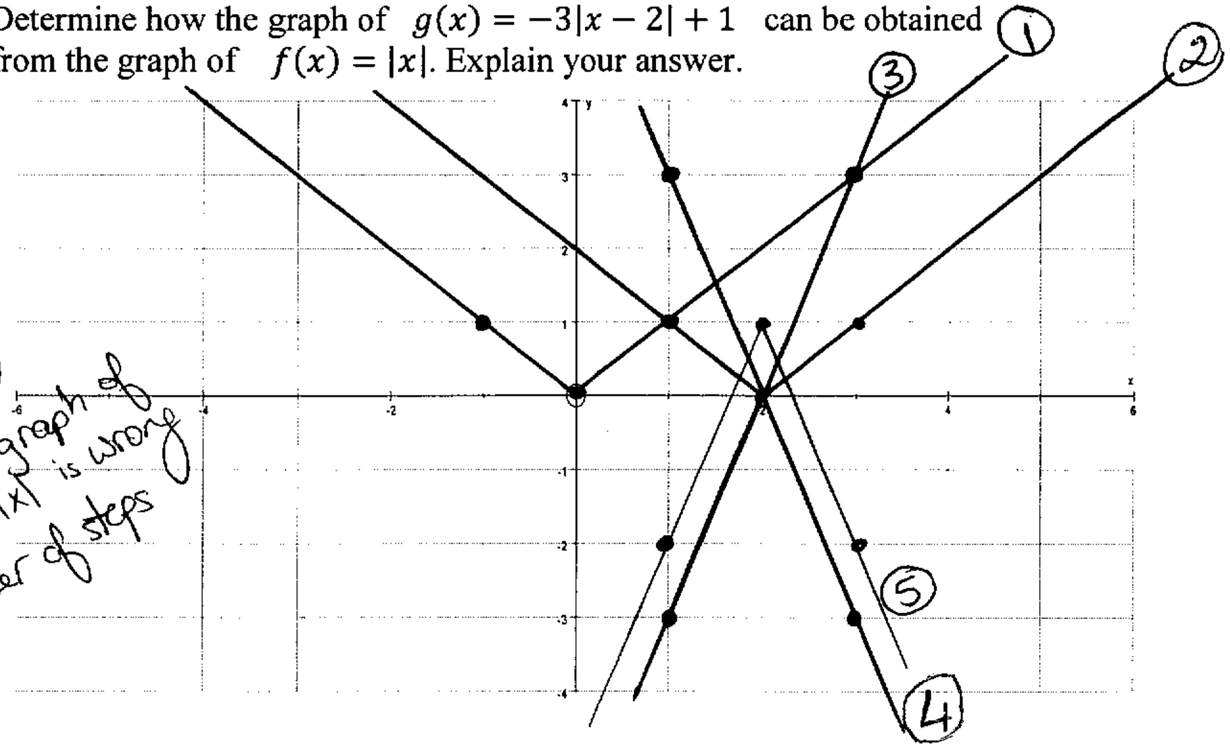
$f(x) = x^5 - 1$

$f(-x)$ 1 $= (-x)^5 - 1$ 1 $= -x^5 - 1$ 1 $\neq f(x)$ not even 1

$\neq -f(x)$ not odd 1

neither

II. (15 pts.)
Determine how the graph of $g(x) = -3|x-2|+1$ can be obtained from the graph of $f(x) = |x|$. Explain your answer.



5

-2 if the graph of $y=|x|$ is wrong

order of steps $-1\frac{1}{2}$

1. $y = |x|$
2. $y = |x-2|$ (1) horizontal shift ($\frac{1}{2}$) 2 units ($\frac{1}{2}$) to the right ($\frac{1}{2}$)
3. $y = 3|x-2|$ (1) vertical ($\frac{1}{2}$) stretch ($\frac{1}{2}$) by a factor 3 ($\frac{1}{2}$)
4. $y = -3|x-2|$ (1) reflect with respect to x axis ($1\frac{1}{2}$)
5. $y = -3|x-2|+1$ (1) vertical shift ($\frac{1}{2}$) 1 unit ($\frac{1}{2}$) up ($\frac{1}{2}$)

$y=\lvert x\rvert$	$y=\lvert x-2\rvert$	$y=3\lvert x-2\rvert$	$y=-3\lvert x-2\rvert$	$y=-3\lvert x-2\rvert+1$
(-1,1)	(1,1)	(1,3)	(1,-3)	(1,-2)
(0,0)	(2,0)	(2,0)	(2,0)	(2,1)
(1,1)	(3,1)	(3,3)	(3,-3)	(3,-2)

add 2 to every x

Multiply every y by 3

Multiply every y by (-1)

add 1 to every y

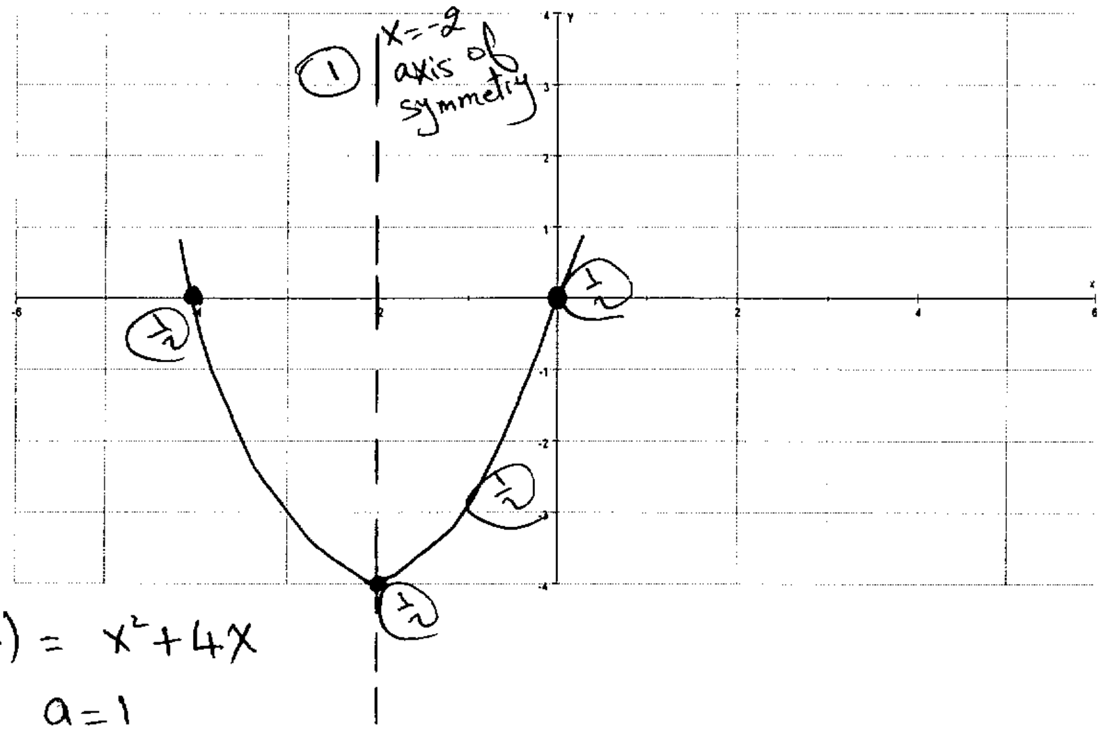
III. Consider the quadratic function $y = f(x) = x^2 + 4x$.

a) (1 pt.) Determine whether the parabola opens up or down.

b) (4 pts.) Find the coordinates of the vertex V and write the equation of the axis of symmetry.

c) (2 pts.) Find the x-intercepts (if they exist) and the y-intercept.

d) (3 pts.) Plot the graphs of the parabola and its axis of symmetry.



$f(x) = x^2 + 4x$

$a = 1$

$b = 4$

$c = 0$

a) $a = 1 > 0$ (1/2) opens up (1/2)

b) $x_v = \frac{-b}{2a}$ (1) $= \frac{-4}{2(1)} = \frac{-4}{2} = -2$ (1/2)

$y_v = f(-2) = (-2)^2 + 4(-2) = 4 - 8 = -4$ (1/2)

Vertex $(-2, -4)$ (1)

Axis of symmetry $x = -2$ (1)

c) x-intercept (1/4)

$y = 0$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$x = 0$ $x = -4$

$(0,0)$ (1/2) $(-4,0)$ (1/2)

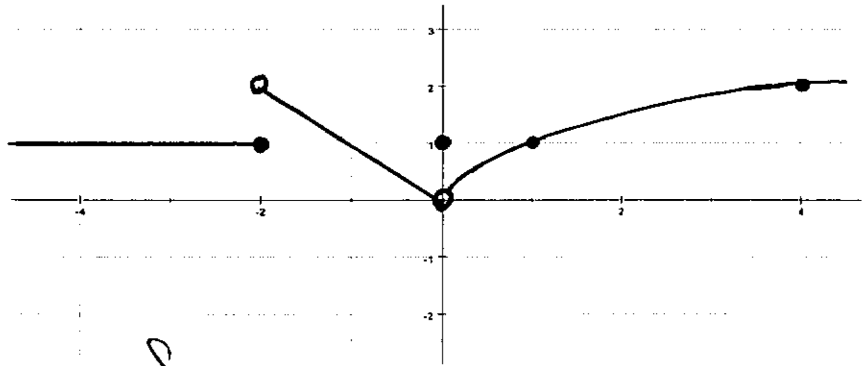
y-intercept (1/4)

$x = 0$

$y = 0$

$(0,0)$ (1/2)

IV. (10 pts.)
Find a formula for the graphed piece-wise function



$$f(x) = \begin{cases} 1 & x \leq -2 \\ -x & -2 < x < 0 \\ 1 & x = 0 \\ \sqrt{x} & x > 0 \end{cases}$$

$1\frac{1}{2}$ each 1 each

V. Without using your calculator, find

a) (3 pts.) $\tan\left(\pi - \frac{\pi}{3}\right) = \frac{\sin(\pi - \pi/3)}{\cos(\pi - \pi/3)}$ 1 $= \frac{\sin \pi/3}{-\cos \pi/3}$ 1 $= \frac{\sqrt{3}/2}{-1/2}$ 1/2 $= -\sqrt{3}$ 1/2

b) (3 pts.) $\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = \cos \frac{\pi}{3}$ 2 $= \frac{1}{2}$ 1

c) (3 pts.) $\cot\left(\frac{10\pi}{3}\right) = \frac{\cos\left(\frac{10\pi}{3}\right)}{\sin\left(\frac{10\pi}{3}\right)}$ 1/2 $= \frac{\cos\left(\frac{9\pi+\pi}{3}\right)}{\sin\left(\frac{9\pi+\pi}{3}\right)}$ 1/2 $= \frac{\cos\left(\pi + \frac{\pi}{3}\right)}{\sin\left(\pi + \frac{\pi}{3}\right)}$ 1/2

$= \frac{-\cos \pi/3}{-\sin \pi/3}$ 1/2

$= \frac{1/2}{\sqrt{3}/2}$ 1/2 $= \frac{1}{\sqrt{3}}$ 1/2 $= \frac{\sqrt{3}}{3}$

d) (3 pts.) $\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$

$= \cos \frac{\pi}{6} \cos\left(-\frac{\pi}{4}\right) - \sin \frac{\pi}{6} \sin\left(-\frac{\pi}{4}\right)$ $1\frac{1}{2}$

$= \frac{\sqrt{3}}{2} \cos \frac{\pi}{4} - \frac{1}{2}\left(-\sin \frac{\pi}{4}\right)$ 1

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$ 1/2

e) (3 pts.) $\sec^2 \frac{\pi}{12}$

$= \frac{1}{\cos^2 \frac{\pi}{12}}$ 1 $= \frac{1}{\frac{1 + \cos \frac{2\pi}{12}}{2}}$ 1 $= \frac{2}{1 + \cos \frac{\pi}{6}}$ $\frac{1}{2}$ $= \frac{2}{1 + \frac{\sqrt{3}}{2}}$ $\frac{1}{2}$ $= \frac{4}{2+\sqrt{3}}$